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Optimization algorithm for rectangular packing based on least wasted area

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Abstract. In view of the problems in solving rectangular packing based on the lowest horizontal line method, the least wasted area method was proposed. The least wasted area method takes reducing the wasted area of the sheet as the goal, and then searches the optimal solution. To improve the efficiency of the algorithm and increase the searching space of solution, the ant colony algorithm was put forward. The rules of updating pheromone, the rules of selecting nodes and the steps of packing algorithm were designed. Test cases show that the proposed algorithm can effectively solve the rectangular packing problem. It performs particularly better for large instances.

Key words. least wasted area, ant colony algorithm, rectangular packing, optimization algorithm.

1. Introduction

The rectangular strip-packing problem is to pack a number of rectangles into a sheet of given width and infinite height so as to minimize the required height. The packing problem is faced in many industries, such as sheet metal cutting, furniture production, glass cutting, newspaper layout, and so on. The different packing has different utilization ratio of sheet. So how to maximize the utilization ratio of the sheet is the goal of the packing. The rectangular packing problem is a kind of special combination optimization problem. In theory, the problem is NP-hard problem. With the increase of the sample, the solution space and the computational complexity are exponentially increasing, it is difficult to obtain the optimal solution of the problem in a short time. Therefore, it is the focus of such research that how

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to design and construct an effective packing optimization algorithm and find a balance between calculation time and material utilization. It has important theoretical significance and application value.

Before the 1990s, a series of packing algorithms were developed, including the BL algorithm [1, 2], BLF algorithm[3], lower step method [4], lowest horizontal line method, improved lowest horizontal line method, DP algorithm [5], DPH algorithm, height adjustment method, remaining rectangle matching method, residual rectangle dynamic matching method, and so on. After the 1990s, some intelligent optimization algorithms (such as genetic algorithms, simulated annealing algorithm, etc.) had become more sophisticated and were successfully applied to resolve the TSP, task scheduling, space allocation, and other combinatorial optimization problems. Therefore, the rectangular packing problem were mostly resolved based on large-scale search algorithm[6, 7].

In this paper, an optimization algorithm for rectangular packing based on least wasted area is proposed. The algorithm has the following characteristics:Considering the pairing between plates and parts, the traditional lowest horizontal line strategy is improved, and the least wasted area strategy is put forward, which effectively improves the utilization rate of the sheet; The ant colony algorithm is improved to solve the problem of premature convergence and inefficient search.

2. Problem description and mathematical model

Rectangular packing problem has different representation in different production practices. According to the size of sheet, the problem includes two kinds: packing on the sheet of given width and given height, and packing on the sheet of given width and infinite height. According to the defect of sheet, the problem includes another two kinds: packing on the non-defective sheet, and packing on the defective sheet. According to the guillotine constraint, the problem includes non-guillotine packing and guillotine packing.

In this paper, the problem of packing rectangles into a sheet of given width and infinite height is considered. The problem can be described as: Given a rectangular sheet of given width and a set of rectangles with various sizes, the strip packing problem of rectangles is to pack each rectangle into the sheet so that no two rectangles overlap and the used sheet height is minimized. In order to describe the problem and optimize the calculation, a mathematical model is established:

The widthof the sheet is W, the number of rectangles is n, the set of rectangles is $R = \{R_1, R_{2,} \ldots R_n\}$, l_i, w_i is respectively the length and width of therectangle R_i(1 $\leq$ i $\leq$ n).(x_{bi}, y_{bi}),(x_{ui}, y_{ui}) is respectively the coordinate of the left bottom corner and the right top corner of therectangle R_i. In order to make two rectangles$R_i$and$R_j$do not overlap, one of the following four conditions must be satisfied: ??$x_{ui} \leq x_{bj}$ $x_{bi} \geq x_{uj}$ $y_{bi} \geq y_{uj}$ $y_{ui} \leq y_{bj}$. In order to ensure that the rectangle does not exceed the boundary of the sheet, then the rectangle R_i must satisfy the following condition: $0 \leq x_{bi} \leq x_{ui} \leq W, 0 \leq y_{bi} \leq y_{ui}$. H is the height set of the outer horizontal contours formed by the packed rectangles, H= $\{H_1, H_{2,} \ldots H_k\}$. The value of k will change dynamically with the packing process.$H_k$will be gradually increased in the set. $H_{max}$is the maximum height in the set. The result of packing is evaluated by the utilization ratio of sheet. U is the utilization ratio of sheet, $U = \int_{i=1}^{n} l_i w_i / H_{max} W$. So the optimal solution of packing problem is the solution that the value of U is the maximum.

3. Rectangular packing strategy based on least wasted area

The traditional lowest horizontal line method only considers packing parts on the lowest horizontal contour line. Although the previousresearchers considered the matching of the part and the contour line, they did not effectively consider that the size and placement of part have impact on the utilization ratio of sheet during the packing process. Therefore, this paper proposes to use the least wasted area strategy to improve it.

In order to avoid the occurrence of small "narrow strip" in some cases and decrease the utilization ratio of the sheet, we introduce the heuristic criteria to select the rectangular part and its location, and determine the final packing by the principle of least wasted area. The flow of judging method is shown in figure 1.

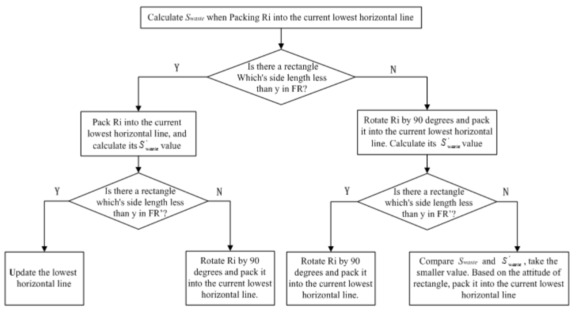

Fig. 1. Rectangularpacking strategy based on least wasted area

In Figure 1, FR represents a closed rectangular set which is formed by four extension lines of rectangularR_i, the current packingfigure and the boundary of sheet. $FR = \{FR_1, FR_{2,} \dots FR_{mount}\} mount$ is the number of closed rectangles.FR' represents the set of closed rectangles obtained by rotating the rectangle $R_i$by 90.S_{waste}, S'_{waste} represents respectively the area of all closed rectangles, and its value can be selected according to the sizes of the rectangles and W. For example:In Figure 2, the dimension of rectangle A is 2018, and the dimension of rectangle B is 2026. The width of sheet is 45, and the height of sheet is infinite. The value of y is 11. When rectangle A has been packed, we begin to pack rectangle B. According to the principle of the lowest horizontal line, the figure when rectangle B is packed is shown in Figure 2-a. The length of short side of left closed rectangle is 8, and the length of short side of right closed rectangle is 5 (as shown in the shadow). According to the principle of least wasted area, this packing result is discarded. So we rotate rectangle B by 90 degrees and pack it into the sheet, as shown in Figure 2-b. But, the boundary of sheet is exceeded. We raise the lowest horizontal line and pack rectangle B, as shown in Figure 2-c. The area of the closed rectangles(wasted area) is 1100. If we rotate rectangle B by 90 degrees and pack it into the sheet (as shown in Figure 2-d), the wasted area is 830. Comparing the wasted area of two results, the smaller value is 830. So the result of figure 2-d is better.

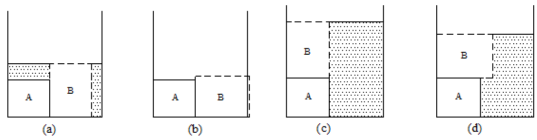

Fig. 2. Example ofleast wasted area strategy

The strategy of leastwasted area proposed in this paper can decide the posture and position of rectangle by the heuristic judgment condition, and avoid the occurrence of small stripduring the packing process by setting the value of thresholdy. In addition, the quality of the result is also considered by the closed rectangular area.

4. Optimization of Rectangular Packing based on Ant Colony Algorithm

In order to improve the efficiency of the algorithm and increase the search space of the solution, the ant colony algorithm is used to optimize the rectangular packing in this paper. The basic idea of the algorithm is: An ant is randomly placed on a rectangle and every rectangle is endowed with pheromone. After the ant accesses all the rectangles without repetition according to a certain sequence, one packing process is completed. If there are m ants, then the number of packing results is m. After completing a round of packing, the pheromone will be updated based on the certain rules and the next round of search will be proceeded until the algorithm ends.

4.1. Rules of updating pheromone

In the ant colony algorithm, the ant will leave pheromone on the path so that the other ants can find the path. The packing order from rectangle R_i to rectangle R_j is denoted as nodev_{ij}. When the ant moves among the nodes, it will leave different pheromone on each node. $\tau_{ij}(t)$isthe pheromone on the node v_{ij} at time t. At the beginning of packing, we can set $\tau_{ij}(0)$to a certain value. When all the ants complete iteration, the pheromone $\tau_{ij}(t)$ will be updated.

According to the idea of Max-Min Ant System (MMAS), it is necessary to strengthen the use of the optimal solution when the pheromone is updated. That is to say, the pheromone of the node of the optimal solution (the global optimal solution or local optimal solution) should be enhanced. The formula of updating pheromoneis shown in (1).

$$\tau_{ij}\left(t+1\right)=\left(1-\rho\right)\tau_{ij}\left(t\right)+\triangle\tau_{ij} \tag{1}$$

In the formula, $\rho \in (0,1)$ denotes the volatilization coefficient of pheromone on the node, $1-\rho$ denotes the persistence coefficient of pheromone, and $\triangle\tau_{ij}$ denotes the increment of pheromone on the node v_{ij}. $\triangle\tau_{ij}$should be able to reflect the merits of the current solution. Thus, according to the objective function of the problem, the expression of $\triangle\tau_{ij}$ is shown in (2).

$$\triangle\tau_{ij} = \begin{cases} \mathrm{Q.U_{best}??v_{ij}} \text{ is included in the global optimal path} \\ \mathrm{Q.U'_{best}}, \quad \mathrm{v_{ij}} \text{ is included in the local optimal path} \\ \qquad\qquad 0\text{other} \end{cases} \tag{2}$$

In this formula, Q-constant, U_{best}- Utilization ratio of sheet in the global optimal path,U'_{best} - The utilization ratio of sheet in the local optimal path.

Strengthening the optimal solution is easy to make the pheromone too large or too small. Aiming at the problem, according to the MMAS algorithm, the pheromone concentration of each node is limited to$[\tau_{min}, \tau_{max}]$, and the value beyond the range is forcibly set to τ_{min}orτ_{max}. This method makes it easy to expand the search range of the ant colony and increase the probability of finding the optimal solution.

4.2. Rules of selecting nodes

The ant uses the selection rule based on pseudo-random ratio to independently select the next level node according to the pheromone and heuristic element. The rule is shown in (3).

$$v_{ij} = \begin{cases} \arg\max_{s\in allowed_k}[(\tau_{sj})^{\alpha} * (\eta_{sj})^{\beta}] & , \quad q \le q_0 \\ S & , \quad otherwise \end{cases} \tag{3}$$

In this formula,$\tau_{ij}$is the pheromone of thenodev$_{\mathrm{ij}}$,η_{ij} is the heuristic elementof the node $\mathrm{v_{ij}}$,$\eta_{ij} = (S_{R_j} - S_{waste_j})/S_{R_j}$. $S_{R_j}$is the area of sheet after the rectangle R_j is packed.*?? and??*is respectively the pheromone factor and the heuristic factor. $allowed_k$is the current set of selectable nodes of the ant k. q is a random number evenly distributed between [0,1]. $q_0 \in (0,1)$is a constant, and it determines the weights of experience and exploration in searching the path.

When an ant is selecting the nodes,q is generated randomly. If $\mathrm{q} \le \mathrm{q_0}$, the next node is selected by equation (3). If $\mathrm{q} \ge q_0$, the value of the probability p_{ij}^k is calculated by equation (4) and the next node is selected by the roulette strategy.

$$p_{ij}^k = \begin{cases} \frac{(\tau_{ij})^{\alpha} * (\eta_{ij})^{\beta}}{\int_{s\in allowed_k}[(\tau_{sj})^{\alpha} * (\eta_{sj})^{\beta}]} & , \quad if\ i \in allowed_k \\ 0 & , \quad otherwise \end{cases} \tag{4}$$

4.3. Steps of rectangular packing based on ant colony algorithm

Step1. Given m ants, place them randomly on the rectangular pieces. The ant on the rectangle R_i begins to pack by the strategy of least wasted area and updates the contour set H. Then the rectangle $R_i$is removed from $allowed_k$. $\mathrm{r}=1$, $U_{best}=0$.

Step2.The next rectangle R_j is selected by the rules of selecting nodes, and R_j

is packed by the strategy of least wasted area. The contour set $H$is also updated. Then the rectangle $R_j$is removed from $allowed_k$ and set the variable$r = r + 1$.

Step3.Judge whether r is equal to n. if r is equal to n, then generate a packing scheme. If r is not equal to n, repeat thesecond step. This way can generate m kinds of packing scheme. Calculate the utilization ratio of sheet of each packing scheme, and save the maximum value as$U^{'}_{best}$.

Step4. Take the global optimalutilization ratio of sheet as $U_{best} = \max(U_{best}, U^{'}_{best})$, and update pheromone according to formulas (1) and (2).

Step5. Repeat step1 to step4. When the number of iterationis equal to the maximum limited number of iteration, the calculation ends.

5. Computational results

According to the relevant literature of ant colony algorithm, and after a number of tests, the parameters of the algorithm are set as follows: $allowed_k$= $(1, 2, \ldots n)$ $m = 1.5n$ $\tau_{min} = 0.6$ $\tau_{max} = 1$ $\tau_{ij}\,(0) = \tau_{max} = 1$ $\alpha = 1$ $\beta = 0.5$ $\rho = 0.4$ $q_0 = 0.9$NC= 20 $Q = 0.5$. The algorithm has been implemented in C # language and is named OALAA algorithm.

In order to test the performance and effect of OALAA algorithm, we choose some classic instances of rectangularpacking, including N and CX.N and CX are zero-waste instances,and their optimal solutions are known. We compare OALAA withalgorithms which are currently believed to be excellent (SVC[8], GRASP[9]). The results are shown in Table 1 - Table 2.

The meaning of the symbols in the table is as follows: n-the number of rectangles included in the instance;W-the width of rectangular sheet; LB-the theoretical minimum height of the packing (theoretical optimal solution); mean-the average solution of the algorithm;best-the optimal solution of the algorithm;Gap%-the relative difference between the mean solution and the theoretical optimal solution, where $\mathrm{Gap} = 100 \times (\mathrm{mean} - \mathrm{LB})/\mathrm{LB}$; Ave.-The average of the values in each column.

Table 1. Computational results of the data set N

Instance	n	W	LB	SVC	GRASP		OALAA	
				mean	best	mean	best	mean
N1	10	40	40	40	40	40	40	40.0
N2	20	30	50	50	50	50	50	50.0
N3	30	30	50	50	51	51	51	50.3
N4	40	80	80	81	81	81	81	81.0
N5	50	100	100	101	102	102	101	101.6
N6	60	50	100	101	101	101	101	101.0
N7	70	80	100	101	101	101	101	101.0
N8	80	100	80	81	81	81	81	81.0
N9	100	50	150	151	151	151	151	151.0
N10	200	70	150	151	151	151	150	150.1
N11	300	70	150	151	151	151	150	150.1
N12	500	100	300	301	304	304	301	301.2
N13	3152	640	960	963	965	965	961	961.5
Ave.				178.62	179.15	179.15	178.38	178.45

Table 2. Computational results of the data set CX

Instance	n	W	LB	SVC	GRASP		OALAA	
				mean	best	mean	best	mean
50cx	50	400	600	603	613	613	613	613.2
100cx	100	400	600	616	617	617	615	618.7
500cx	500	400	600	604	605	605	600	600.0
1000cx	1000	400	600	601	602	602	600	600.0
5000cx	5000	400	600	600	600	600	600	600.0
10000cx	10000	400	600	600	600	600	600	600.0
15000cx	15000	400	600	600	600	600	600	600.0
Ave.				603.4	605.29	605.29	604.00	604.56

By comparison of the algorithm, for the instance N in Table 1, SVC can get three optimal solutions, GRASP can get two optimal solutions, while OALAA can get four optimal solutions. Gap% value of OALAA is 0.62, and itis significantly lower than the SVC and GRASP; For the instance CX in Table 2, SVC can get three optimal solutions, GRASP can get three optimal solutions, while OALAA can get five optimal solution; Gap% value of OALAA is 0.8, and it is higher than SVC , but lower than GRASP.It can be seen that OALAA algorithm proposed in this paper has some advantages for solving the problem of rectangularpacking.

6. Conclusion

In this paper, the traditional method of rectangular packing based on the principle of the lowest horizontal lineis improved, and thestrategy based on the least wastedarea is put forward. Using the heuristic criteria to select the rectangular parts and their locations can effectively utilize the region of sheet and avoid the waste of sheet. The Computational results show thatthe optimization algorithm proposed in this paper can effectively solve the problem of rectangular packing, especially for the large instances.

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